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M BROWN JUN 83 AFOSR-TR-83-0820 AFOSR-82-0024

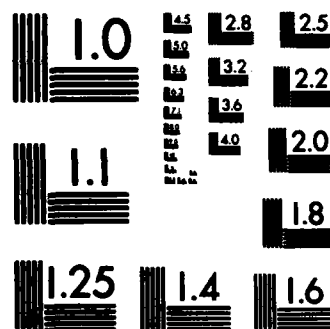
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) This report documents progress achieved during the period of the grant. Excellent bounds were obtained for the goodness of exponential approximation for the distribution of time to first failure starting with all components functioning, as reported in the paper, "On the Reliability of Repairable Systems." The technical report, "Exponential Approximations for Two Classes of Aging Distributions" continues the principal investigator's work into the topic of exponential approximations. Documentation of other papers produced include, "A Measure of Variability Based on the Harmonic Mean, and its Use in Approximations;" (CONT.)		

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ITEM #20, CONT.: "IFR Results for Repairable Systems;" a revised version of "Approximating DFR Distributions by Exponential Distributions, with Applications to First Passage Times;" "Imperfect Maintenance;" and "Imperfect Repair."

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**Final Scientific Report to Air Force
Office of Scientific Research**

Grant AFOSR-82-0024

**On the Reliability of Systems Subject
to Maintenance and Repair**

**Department of Mathematics
The City College, CUNY
New York, NY 10031**

Mark Brown, Principal Investigator

June, 1983

**Approved for public release;
distribution unlimited.**

Grant AFOSR-82-0024 was awarded for the period 12/1/81-11/30/82. It was subsequently extended without further funding until 4/30/83. This report will discuss the research accomplishments achieved during the grant period.

1) A technical report "On the Reliability of Repairable Systems" (AFOSR-82-0024 Technical Report No. 82-01, City College, CUNY Technical Report No. MB1) was issued in October 1982. The paper will appear in a special issue of Operations Research devoted to Reliability Theory, scheduled for summer 1983.

The paper achieves an important objective of the proposal. Excellent bounds are obtained for the goodness of exponential approximation for the distribution of time to first failure starting with all components functioning. This distribution has been studied by many authors, including both engineers and mathematicians. The current results are by far the best yet achieved.

2) A technical report "Exponential Approximations for Two Classes of Aging Distributions" by Mark Brown and Guangping Ge (AFOSR-82-0024 Technical Report No. 82-02, City College, CUNY Technical Report No. MB2) was issued in February 1983.

Guangping Ge is a Professor from the People's Republic of China. He was a visiting Professor at the CUNY Graduate Center and is now a Visiting Professor at Berkeley.

The report continues my research (began in Brown [II-1]) into the topic of exponential approximations. The main result is that if

F is NBUE (new better than used in expectation), and G is the stationary renewal distribution corresponding to F then:

$$(1) \quad \sup |\bar{F}(t) - \bar{G}(t)| \leq A\rho^{1/2}$$

$$(2) \quad \sup |\bar{F}(t) - e^{-t\mu}{}^{-1}| \leq A\rho^{1/2}$$

$$(3) \quad \sup |G(B) - \int_B \mu^{-1} e^{-t\mu}{}^{-1} dt| \leq \rho$$

$$(4) \quad \sup |\bar{G}(t) - e^{-t\mu_G}{}^{-1}| \leq \rho$$

where $\bar{F} = 1-F$, $\bar{G} = 1-G$, $A = \frac{4\sqrt{6}}{\pi} \approx 3.119$, $\mu = E_F X$, $\mu_2 = E_F X^2$,

$\mu_G = E_G X = \frac{\mu_2}{2\mu}$ and $\rho = \left| \frac{\mu_2}{2\mu^2} - 1 \right| = 1 - \frac{\mu_G}{\mu}$. Thus if F is NBUE

and ρ is small then F and G are approximately equal and approximately exponentially distributed. A similar result is shown to hold for F NWUE (new worse than used in expectation). Since these are the weakest of the commonly studied aging properties, random variables with NBUE or NWUE distributions often arise in applications. The above approximations are thus quite useful.

The best potential improvement in (2), for bounds of the form $c\rho^\alpha$, is the lowering of c from 3.119 to 1. The exponent $\alpha = \frac{1}{2}$ is the best possible. This conclusion remains true even within the subclass of IFRA (increasing failure rate on the average distributions).

It also follows that if $\{X_n\}$ is a sequence of NBUE distributions, then $\frac{X_n}{EX_n}$ converges in distribution to an exponential distribution

if and only if

$$\lim_{n \rightarrow \infty} \rho_n = \lim_{n \rightarrow \infty} \left[1 - \frac{EX_n^2}{2(EX_n)^2} \right] = 0 ,$$

in which case the mean of the limiting exponential distribution equals 1. This result is shown by counterexample not to hold for F NWUE.

Finally if $Y = \sum_1^N X_i$ where $\{X_i, i \geq 1\}$ is an i.i.d. sequence of random variables with NBUE distribution F, and N is geometrically distributed with parameter p independent of $\{X_i\}$, then:

$$\sup |\Pr(Y > t) - e^{-\mu t}| \leq A(p\rho)^{1/2}$$

where $A = \frac{4\sqrt{6}}{\pi}$ and $\rho = 1 - \frac{EX^2}{2(EX)^2}$. A similar result holds for NWUE.

This report has been submitted for publication to the Annals of Probability.

3) A technical report "A Measure of Variability Based on the Harmonic Mean, and its Use in Approximations" (AFOSR-82-0024 Technical Report No. 82-03, City College, CUNY Technical Report No. MB3) was issued in March 1983.

The main result is that if F is a distribution on $[0, \infty)$ with $\mu = E_F X$, $a = E_F X^{-1}$ and $c^2 = 1 - (\mu a)^{-1}$ then for g completely monotone:

$$(5) \quad 0 \leq E g(X) - g(EX) \leq c^2 g(0)$$

$$(6) \quad \text{Var}(g(X)) \leq c^2 g^2(0) .$$

The import of (5) and (6) is that if c^2 is small then $\frac{g(X)}{g(0)}$ is heavily concentrated around $\frac{g(EX)}{g(0)}$, for g completely monotone. In that sense c^2 can be thought of as a measure of variability for F , which is reflected in the behavior of $g(X)$ for g completely monotone.

Comparing c^2 with σ^2 we see some similarities. Jensen's inequality yields both $EX^2 \geq (EX)^2$ and $EX^{-1} \geq (EX)^{-1}$. The quantity σ^2 is a measure of the discrepancy between EX^2 and $(EX)^2$ while c^2 is a scale invariant measure of the discrepancy between EX^{-1} and $(EX)^{-1}$. Note that σ^2 is heavily influenced by the tail of F but that the behavior of $g(X)$ for g completely monotone (and thus rapidly decreasing) is not sensitive to the tail behavior of F . Thus σ^2 should not be informative about the degree of clustering of $g(X)$ around $g(EX)$.

Finally Chebichev's inequality can be interpreted as:

$$(7) \quad 0 \leq E g(X) - g(EX) \leq \frac{\sigma^2}{a^2}$$

where

$$g(x) = \begin{cases} 1 & \text{if } |x - \mu| \geq a \\ 0 & \text{if } |x - \mu| < a \end{cases}$$

Thus (5) is analogous to Chebichev's inequality (7); (5) involves the measure of variability c^2 while (7) involves σ^2 , and (5) applies to the class of completely monotone functions while (7) applies to a class of indicator functions.

I believe that c^2 is a natural and potentially useful new measure of variability.

This report has been submitted for publication to the Annals of Statistics.

4) A technical report "IFR Results for Repairable Systems" by N.R. Chaganty (AFOSR-82-0024 Technical Report No. 82-04, City College, CUNY Technical Report No. MB4) was issued in March 1983.

Professor Chaganty is an Assistant Professor in the Department of Mathematical Sciences at Old Dominion University. He served as a Consultant on the grant. Chaganty and I have collaborated in research on first passage time distributions in reliability (Brown and Chaganty [II-2]) and have a continuing interest in this area.

The report considers a k-out-of-n system with independent repairable components. It is shown that if the difference between failure and repair rates is the same for each component then the time to first failure is IFR (increasing failure rate). This generalizes a result previously derived by Brown and Chaganty [II-2].

The general problem is to determine whether or not the time to first failure for a coherent system of independent repairable memoryless components is IFRA. I continue my efforts on this problem. The current report deals with an interesting special case.

The report has been accepted for publication in the Naval Research Logistics Quarterly.

5) In 1979 I wrote a paper "Approximating DFR Distributions by Exponential Distributions, with applications to First Passage Times", under AFOSR Grant No. F49620-79-C-0157. It was accepted for publication by the Annals of Probability in October 1980, but I decided to hold up publication pending an improvement of the main result, which I felt was obtainable. Recently (during the grant period) I was able to achieve the desired result. This gives the best possible upper bound on the sup norm distance between an IMRL distribution and an exponential distribution with the same mean, when the first two moments are known. It greatly improves the bounds obtained by other authors for completely monotone distributions and holds for a considerably larger class of distributions.

The new revised version was again accepted by the Annals of Probability and appeared in the April 1983 issue. I did not issue the paper as a technical report under the current grant since it is a revision of a previous AFOSR report. I did forward a copy of the paper to Dr. Smythe.

6) I wrote a paper with Dr. Frank Proschan, entitled "Imperfect Maintenance". It appeared in the conference proceedings of the conference on Survival Analysis held in Madison, Wisconsin. As Dr. Proschan issued the paper as a Florida State AFOSR technical report, I did not issue it as a technical report under this grant.

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The paper derives inequalities and monotonicity properties for stochastic processes generated by failing items which are sometimes only imperfectly repaired (in a sense defined in the paper). As a by-product we obtain several new results for proportional hazard families of distributions. The problem of imperfect repair is an important one with obvious practical application. My work with Dr. Proschan continues. Meanwhile, a previous effort of ours in this area ("Imperfect Repair," 1980 AFOSR, FSU report) is scheduled to appear in the December 1983 issue of the Journal of Applied Probability.

7) I have achieved some striking inequalities for IFR (increasing failure rate) distributions. To obtain them I had to derive a lower bound for the correlation between an IFR random variable X , and its cumulative hazard function $H(X)$. After unsuccessful effort over a long period of time I recently made an important breakthrough. I plan to write a report which I will issue as an addendum to this project.

I will disclose this problem further in my new proposal which I plan to submit in the near future.

8) In addition to the IFR inequalities mentioned in [7] above, I have partial results on a few more topics. I anticipate that all will lead to good papers in the near future. Two of these are:

(1) I have been working on the properties of failure rate ordering, a partial ordering between probability distributions on

$[0, \infty)$. This interest is motivated by my paper (II-1) discussed above in [5].

(ii) I have been working on the problem of the first occurrence of a specified pattern in multinomial trials. I have found that if a pattern is of length m and has small probability p , then a geometric distribution truncated below at m provides an excellent approximation to the distribution of the waiting time for the first occurrence of the pattern.

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I-Technical reports issued under Grant AFOSR-82-0024.

- [1] "On the Reliability of Repairable Systems," by Mark Brown; October 1982. To appear in a special Reliability issue of Operations Research.
- [2] "Exponential Approximations for Two Classes of Aging Distributions," by Mark Brown and Guangping Ge; February 1983. Submitted for publication to the Annals of Probability.
- [3] "A Measure of Variability Based on the Harmonic Mean and its Use in Approximations,"
- [4] "IFR Results for Repairable Systems," by N.R. Chaganty; March 1983. Accepted for publication to the Naval Research Logistics Quarterly.

II-Previous papers of Mark Brown published during the grant period, in press, or accepted for publication.

- [1] "Approximating IMRL Distributions by Exponential Distributions, with Applications to First Passage Times." The Annals of Probability, 1983, Vol. 11, No. 2, 419-27.
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